

## N O T I C E

THIS DOCUMENT HAS BEEN REPRODUCED FROM  
MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT  
CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED  
IN THE INTEREST OF MAKING AVAILABLE AS MUCH  
INFORMATION AS POSSIBLE

(E81-10112) A NEW METHOD OF CURIE DEPTH  
EVALUATION FROM MAGNETIC DATA: THEORY  
Progress Report (North Carolina State Univ.)  
9 P HC A02/MF A01

881-22448

UNCLAS  
00112

63/43

"Not available under NASA sponsorship  
in the interest of early and wide dis-  
semination of Earth Resources Survey  
Program information and without liability  
for any use made thereof."

Progress Report (NAS5-26157)

to

NASA Goddard Space Flight Center

Greenbelt, MD 20071

on

A New Method of Curie Depth Evaluation  
from Magnetic Data - Theory

Submitted

by

*North Carolina St U*

I. J. Won

Department of Marine, Earth, & Atmospheric Sciences

P. O. Box 5068

Raleigh, N. C. 27650

February 25, 1981

RECEIVED

MAR 5, 1981

SIS/902.6

M-008

TYPE II

8.1-10112  
CR-164106

## Summary

A new approach in estimating the Curie point isotherm is developed by using the classical Gauss method of inverting a system of nonlinear equations. The method, slightly modified by the differential correction technique as described by Won (1981), will be used for a direct inversion of filtered Magsat data to calculate the crustal structure above the Curie depth, which is modelled as a magnetized layer of varying thickness and susceptibility. Since the depth below the layer is assumed to be non-magnetic, the bottom of the layer is interpreted as the Curie depth. The method, once fully developed, tested, and compared with previous work by others, will be applied to a portion of the eastern U.S. when enough amount of Magsat data accumulate in the region.

## Review

The basic objective in relating the aeromagnetic field data with the structure of the Curie point isotherm is to compute the lower depth limit of magnetized masses in the earth's crust. Rocks lose their magnetism at the Curie temperature at which ferrimagnetic rocks become paramagnetic, and their ability to produce detectable magnetization disappears. Thus, the deepest level in the crust containing materials with discernible magnetization is generally interpreted as the depth to the Curie point isotherm.

The Curie point is about 580° C for magnetite. With appropriate titanium substitutions, Buddington and Lindsley (1964) calculated an average Curie point ranging between 520° C and 560° C for rocks in the deep crust. It is generally believed that the amount of geothermal heatflow should correlate with the Curie depth and thus, in turn, to the crustal magnetic field.

Our main goal is, therefore, to determine the bottom shape of the magnetized crust from a magnetic anomaly map. Since the magnetic anomalies attributable to the bottom geometry are usually quite smaller and have much longer wavelengths than those produced by shallow geological variations, the problem is comparable to searching for a needle in a haystack. Early studies include those by Vacquier and Affleck (1941) and Bhattacharyya and Morley (1965). In both cases, each isolated anomaly was filtered and separately interpreted by the empirical graphic method using a vertical-sided prism.

A more sophisticated method was proposed by Bhattacharyya and Leu (1975a, 1975b). Their method requires an extensive initial filtering of the aeromagnetic data in both regional and short wavelength domains. The filtered data is subsequently divided into a large number of blocks. For each block, a two-dimensional spectrum and its moments are computed and compared with a model of an isolated vertical-sided prism within a block in order to locate the corners of the body. The total amount of computation is tremendous since the method requires a two-dimensional Fourier transform for each block. Applying the method to the Yellowstone National Park area, they produced the Curie isotherm map well correlated with the known geothermal area.

Employing a similar technique, Shuey et al. (1977) concluded that it is essentially impossible to determine Curie depth with any resolution at all by fitting a vertical prism to a single anomaly. The Curie depths they derived could be changed by as much as 10 Km without violating the observed data. This conclusion is seemingly in conflict with those of Bhattacharyya and Leu (1975b).

All methods reviewed here are commonly based on the assumption that there exists an isolated magnetic source for each anomaly. Each individual anomaly is assumed to be caused by a single vertical-sided prism (Bhattacharyya and Leu, 1975a, 1975b) or a truncated vertical cone (Shuey et.al., 1977). Such isolated models are apt to generate spurious anomalies, particularly due to their unrealistically well-defined corners and vertical surfaces. These spurious anomalies can induce significant errors in either direct-modelling or spectrum calculation.

Rock formations causing long wavelength magnetic anomalies at a depth close to the Curie point are more likely to have a continuous lateral distribution rather than isolated blocks of well-defined geometrical bodies. A realistic model at this depth should manifest a continuous lateral distribution of magnetic materials having variable thicknesses and susceptibilities.

Fluctuations in long wavelength magnetic anomalies can be attributed to lateral variations either in magnetization strength or in Curie depth. These double uncertainties make the task of simultaneously determining both the magnetizations and the Curie depths very difficult, if not impossible. Similar uncertainties apply to many geophysical modelling theories, e.g., a thin magnetic dike for which the anomaly is the same as long as the product of thickness and susceptibility remains the same. However, it can be shown that the statement is no longer true if the dike has a considerable thickness for which case both the thickness and the susceptibility can be independently determined from observed data (Won, 1981). The following treatment is based on the classical Gauss method for solving non-linear equations (Corbato, 1965; Johnson, 1969) modified for digital total field magnetic data (Won, 1981) in order to invert regional aeromagnetic data for a continuous crustal mass of varying thicknesses and susceptibilities.

### Proposed Method

#### 1. Mathematical Model for the Curie Depth

Figure 1 shows the model which will be used for inverting aeromagnetic data. The model consists of laminated thick vertical prisms having flat top surfaces and linearly-connected inclined bottom surfaces. The magnetic susceptibility below the lower boundary is assumed to be zero so that the bottom geometry represents the Curie isotherm topography. Although data will be confined within the laminated block region, two semi-infinite slabs are added on either side in order to reduce the edge effects of the first and last blocks. The unknown parameters to be determined are the depth ( $h$ 's) at each nodal point and the magnetic susceptibility ( $k$ 's) of each prismatic body.

The model is two-dimensional with an arbitrary strike angle with respect to the magnetic north. Data are assumed to be obtained at a constant altitude along a traverse perpendicular to the strike. Since the method uses total field aeromagnetic data, there is no need for reducing the data to the polar anomalies.

#### 2. Mathematical Analysis

Won (1981) formulated an inversion method for an inclined dike of arbitrary thickness, depth of burial, dip angle, and susceptibility by

employing Gauss' method, as well as the differential correction technique. Starting from a set of initially guessed parameters, the inversion method self-corrects these parameters iteratively until the observed data fits best to the model.

By combining two inclined dikes, we can derive a total field magnetic anomaly formula for a single vertical block having a flat top and an inclined bottom. After some involving manipulation, we can derive the following formula for the entire laminated prismatic bodies as shown in Figure 1:

$$\frac{F(X)}{2F_0} = \sum_{j=1}^{N-1} k_j [Aq_j + Bp_j - S_j^2(A-B\Delta t_j)q_j + (A\Delta t_j + B)P_j] + k_1 (A\bar{q}_0 - B\bar{p}_0) + k_{N-1} (A\bar{q}_N + B\bar{p}_N) \quad (1)$$

$F(X)$  denotes the total magnetic field anomaly measured at a distance  $X$  with respect to the origin. The first term is the total contribution of  $N$  laminated bodies, while the second and third terms represent the contribution of the left semi-infinite slab and the right semi-infinite slab, respectively. Table 1 shows the mathematical notations used in equation (1). The formula can be used for a forward calculation of the total field anomaly for any given set of variations in depths and susceptibilities.

### 3. Inversion of Anomaly by Gauss' Method and Differential Correction Technique.

The unknown parameters include the depth profile ( $h$ 's) and the lateral magnetic variation ( $k$ 's) which are to be determined from a given profile obtained along a line perpendicular to the strike. For  $N$  laminated blocks, there are  $(2N-1)$  unknowns. All other quantities appearing in equation (1) are considered known.

Suppose that the approximate values for  $h$ 's and  $k$ 's are assumed or otherwise established, even though the values may be significantly in error. Following a similar approach by Won (1981), we shall describe a method of simultaneously improving all unknown parameters by using a technique combining the differential correction and the non-linear least-squares method.

Let us write  $F(X)$  in equation (1) as

$$F(X) = F(X; h_1, h_2, \dots, h_N; k_1, k_2, \dots, k_{N-1}). \quad (2)$$

If we now change  $h_j$  ( $j = 1, 2, \dots, N$ ) and  $k_j$  ( $j = 1, 2, \dots, N-1$ ) by small amounts  $\Delta h_j$  and  $\Delta k_j$ , respectively, the magnetic anomaly  $F(X)$  will change by  $\Delta F$  such that

$$\Delta F = \sum_{j=1}^N \frac{\partial F}{\partial h_j} \Delta h_j + \sum_{j=1}^{N-1} \frac{\partial F}{\partial k_j} \Delta k_j. \quad (3)$$

For  $M$  points of digitized total field magnetic data  $G_i$  ( $i = 1, 2, \dots, M$ ;  $M > 2N - 1$ ), we attempt to minimize the quantity  $S$

$$S = \sum_{i=1}^M (F_i + \Delta F_i - G_i)^2 \quad (4)$$

with respect to  $\Delta h_j$  and  $\Delta k_j$ . Differentiating with respect to  $\Delta h_j$  and  $\Delta k_j$ , we obtain a system of  $(2N-1)$  simultaneous linear equations:

$$\begin{bmatrix} \sum_{k=1}^M \frac{\partial F_k}{\partial h_j} & \frac{\partial F_k}{\partial h_j} & \sum_{k=1}^M \frac{\partial F_k}{\partial k_j} & \frac{\partial F_k}{\partial k_j} \\ \sum_{k=1}^M \frac{\partial F_k}{\partial h_j} & \frac{\partial F_k}{\partial h_j} & \sum_{k=1}^M \frac{\partial F_k}{\partial k_j} & \frac{\partial F_k}{\partial k_j} \end{bmatrix} \begin{bmatrix} \Delta h_j \\ \Delta k_j \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^M \frac{\partial F_k}{\partial h_j} (F_k - G_k) \\ \sum_{k=1}^M \frac{\partial F_k}{\partial k_j} (F_k - G_k) \end{bmatrix} \quad (5)$$

The normal matrix in the first bracket has an order of  $(2N-1)$  and is symmetric and positive-definite. Once we solve equation (5) for the unknown differential correction vector  $[\Delta h_j, \Delta k_j]$ , we obtain the improved values  $h'_j$  and  $k'_j$  such that

$$h'_j = h_j + \Delta h_j, \quad (6A)$$

$$k'_j = k_j + \Delta k_j. \quad (6B)$$

By replacing  $h_j$  and  $k_j$  with  $h'_j$  and  $k'_j$ , respectively, we can iterate the process until the RMS error is minimized.

The expressions for derivatives in equation (5) are rather involving. Let us first define

$$\begin{aligned} \psi_j = \left( \frac{s_j^2}{w} \right)^2 & \left[ (2\Delta h_j A + u_j B) q'_j + (2\Delta h_j B - u_j A) p'_j \right. \\ & \left. + \frac{1}{r_j^2} \{ (A - \Delta t_j B) h'_j - (\Delta t_j A + B) x_j \} \right] \end{aligned} \quad (7A)$$

and

$$\begin{aligned} \psi'_j = \left( \frac{s_{j-1}^2}{w} \right)^2 & \left[ (2\Delta h_{j-1} A + u_{j-1} B) q'_{j-1} + (2\Delta h_{j-1} B - u_{j-1} A) p'_{j-1} \right. \\ & \left. + \frac{1}{r_{j-1}^2} \{ (A - \Delta t_{j-1} B) h'_{j-1} - (\Delta t_{j-1} A + B) x_{j-1} \} \right]. \end{aligned} \quad (7B)$$

Derivatives with respect to  $h_j$  can now be written as

$$\frac{1}{2F_0} \frac{\partial F}{\partial h_j} = k_j \psi_j - k_{j-1} \psi'_j \quad ; \text{ for } j = 2, 3, \dots, (N-1), \quad (8A)$$

$$= k_1 \psi_1 - \frac{k_1}{r_1^2} (A h'_1 - B x_1); \text{ for } j = 1, \quad (8B)$$

$$= -k_{N-1} \psi'_N + \frac{k_{N-1}}{r_N^2} (A h'_N - B x_N); \text{ for } j = N. \quad (8C)$$

Similarly, derivatives with respect to  $k_j$ 's can be written as

$$\frac{1}{2F_0} \frac{\partial F}{\partial k_j} = Aq_j + Bp_j - s_j^2 \{ (A - B\Delta t_j) q_j + (A\Delta t_j + B)p_j \}$$

$$\equiv T_j \quad ; \text{ for } j = 2, 3, \dots, N-1, \quad (9A)$$

$$= T_1 + (A\bar{q}_0 - B\bar{p}_0); \text{ for } j = 1, \quad (9B)$$

$$= T_{N-1} + (A\bar{q}_N + B\bar{p}_N); \text{ for } j = N. \quad (9C)$$

The method described thus far does not necessarily converge to a correct final result: equation (3) is based on the assumption that the values of  $\Delta h_j$ 's and  $\Delta k_j$ 's are small compared with the current values of  $h_j$ 's and  $k_j$ 's. Therefore when the current  $h_j$ 's and  $k_j$ 's are grossly wrong, equation (5) will produce large  $\Delta h_j$ 's and  $\Delta k_j$ 's which may violate the assumption of differential correction and, consequently, the process may not converge. The safest way to avoid such a problem is to impose a certain allowable limit on each parameter. Such limitations may include: i) all depths are positive and less than, say, 100 Km, and ii) all susceptibilities must be within, say, 0 and 0.01 c.g.s.. The conditions can even be tightened with more geological knowledge of the area. If any  $h_j$ 's or  $k_j$ 's go beyond its constraint, it may be allowed to change up to that limit.

Unlike the inversion approaches of individually correcting each unknown parameter by trial-and-error (e.g., McGrath and Hood, 1970) where the convergence to the absolute minimum error state depends on the choice of initial parameters, the present method simultaneously reduces all  $(2N-1)$  parameters regardless of their initial values.

The method once developed will be first tested with the same data in the Yellowstone National Park Area used by Bhattacharyya and Leu (1975b) and the results will be compared critically.

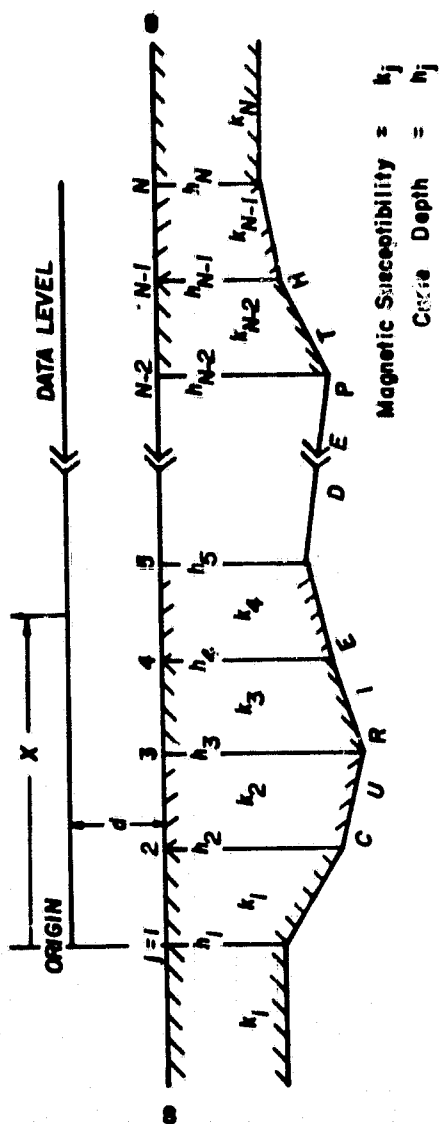


Figure 1. Mathematical model of the Curie depth



Table 1. Mathematical Notations and Abbreviations

NOTATIONS:

N	total number of nodal points
$h_j$	Curie depth at j-th nodal point
$k_j$	magnetic susceptibility of a block between j-th and (j+1)-th nodal points
d	altitude of data (constant)
w	width of block (constant)
$F_0$	the main Earth field in gamma
I	inclination of the main Earth field
$\beta$	strike angle measured CCW from the magnetic north
X	distance measured from origin

ABBREVIATIONS:

$$\begin{aligned}
 A &= \sin \beta \sin 2I & B &= \sin^2 \beta \cos^2 I - \sin^2 I \\
 x_j &= X - (j-1)w & h_j' &= h_j + d \\
 u_j &= (w^2 - \Delta h_j^2)/w & \Delta h_j &= h_j - h_{j+1} \\
 s_j^2 &= w^2 / (w^2 + \Delta h_j^2) & \Delta t_j &= \Delta h_j / w \\
 r_j^2 &= x_j^2 + d^2 & r_j'^2 &= x_j^2 + h_j'^2 \\
 \bar{q}_0 &= \ln(r_1/r_1') & q_j' &= \ln(r_{j+1}'/r_j') \\
 q_j &= \ln(r_{j+1}/r_j) & \bar{q}_N &= \ln(r_N'/r_N) \\
 \bar{p}_0 &= \tan^{-1} \frac{d}{x_j} - \tan^{-1} \frac{h_1'}{x_j} & \bar{p}_N &= \tan^{-1} \frac{d}{x_N} - \tan^{-1} \frac{h_N}{x_N} \\
 p_j &= \tan^{-1} \frac{d}{x_j} - \tan^{-1} \frac{d}{x_{j+1}} \\
 p_j' &= \tan^{-1} \frac{wh_j' - x_j \Delta h_j}{\Delta h_j h_j' + x_j w} - \tan^{-1} \frac{wh_{j+k}' - x_j \Delta h_j}{h_j h_{j+k}' + x_{j+1} w}
 \end{aligned}$$

## References

- Bhattacharyya, B. K., and Leu, L. K., 1975a, Spectral analysis of gravity and magnetic anomalies due to two-dimensional structures, *Geophysics*, V. 40, pp. 993-1013.
- Bhattacharyya, B. K., and Leu, L. K., 1975b, Analysis of magnetic anomalies over Yellowstone National Park: Mapping of Curie point isothermal surface for geothermal reconnaissance, *J. Geophys. Res.* 80, 30, pp. 4461-4465.
- Bhattacharyya, B. K., Morley, L. W., 1965, The lineation of deep crustal magnetic bodies from total field aeromagnetic anomalies, *J. Geomag. Geoelect.*, 17, pp. 237-252.
- Buddington, A. F., and Lindsley, D. H., 1964, Iron-titanium oxide minerals and syntnetic equivalents: *J. Petrology*, v. 6, pp. 310-357.
- Carbato, C. E., 1965, A least-squares procedure for gravity interpretation, *Geophysics*, 30, pp. 228-233.
- Johnson, W. W., 1969, A least-squares method of interpreting magnetic anomalies caused by two-dimensional structures, *Geophysics*, 34, pp. 65-74.
- Shuey, R. T., Schellinger, D. K., Tripp, A. C., and Alley, L. B., 1977a, Curie depth determination for aeromagnetic data, *Geophys. J., R. Astr. Soc.* 50, pp. 75-101.
- Shuey, R. T., Schellinger, D. K., Johnson, E. H., and Alley, L. B., 1977b, Aeromagnetcs and the transition between the Colorado plateau and Basin Range provinces. *Geoph. Journ.* v. 77, pp. 107-110.
- Vacquier, V., Affleck, J., 1941, A computation of the average depth to the bottom of the earth's magnetic crust based on a statistical study of local magnetic anomalies, *Trans. Am. Geophys. Union*, pp. 446-450.
- Won, I. J., 1981, Application of Gauss' method to magnetic anomalies of dipping dikes, *Geophysics*, 46, 2, pp. 211-215.